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of the base mid way between the upper and lower bases ; and a = altitude of frustum. Then $\rho = \frac{1}{2}(R+r)$. $\therefore 4\rho^2 = (R+r)^2$.

Volume of frustum = $\frac{1}{3}\pi a(R^2 + r^2 + Rr) = \frac{1}{3}\pi a(2R^2 + 2r^2 + 2Rr) = \frac{1}{3}\pi a[R^2 + r^2 + (R+r)^2] = \frac{1}{3}\pi a(R^2 + r^2 + 4\rho^2)$.

The same method applies to frustums of pyramids, and all solids coming under the prismatoid formula as special cases.

Also solved in a more general manner by *B. F. SINE, P. S. BERG, J. SCHEFFER, HARVEY N. DAVIS, W. H. DRANE, and CHAS. C. CROSS.*

CALCULUS.

68. Proposed by EDWARD DRAKE ROE, JR., A. M., Associate Professor of Mathematics, Oberlin College, Oberlin, Ohio.

If a^x to r steps be denoted by a^r , and if $y = x^r$, prove that

$$D_x y = x^{r+\frac{r-1}{x}+\dots+\frac{1}{x}} (\log x)^{r-1} (1 + \log x) + \sum_{k=2}^{k=r} x^{r+\frac{r-1}{x}+\dots+\frac{k-1-1}{x}} (\log x)^{r-k}.$$

Solution by the PROPOSER.

If $y = f_1(x)^{f_2(x)}$, we obtain, by taking the logarithm of both sides of the equation, and differentiating, the formula

$$D_x y = f_1(x)^{f_2(x)} \log f_1(x) D_x f_2(x) + f_1(x)^{f_2(x)-1} f_2(x) D_x f_1(x).$$

If in this $f_1(x) = x$, $f_2(x) = x^r$, that is if $y = x^x = x^r$, we obtain

$$D_x y = x^{x+\frac{1}{x}} \log x (1 + \log x) + x^{x+\frac{1}{x}-1},$$

and the formula to be proved is true when $r = 2$. It is evidently not true for values of $r < 2$. Assume that it is true for all other values of r

Let $y = x^x = x^r$. In the above formula put, $f_1(x) = x$, $f_2(x) = x^r$, and we get

$$D_x y = x^x \log x D_x x^r + x^x \sum_{k=2}^{r+1} x^{r+\frac{r-1}{x}+\dots+\frac{k-1-1}{x}} (\log x)^{r-k} x^r,$$

but by this assumption this is

$$\begin{aligned} D_x y &= x^x \log x [x^{x+\frac{r-1}{x}+\dots+\frac{1}{x}} (\log x)^{r-1} (1 + \log x)] + x^x \log x \sum_{k=2}^{r+1} x^{x+\frac{r-1}{x}+\dots+\frac{k-1-1}{x}} (\log x)^{r-k} x^r \\ &\quad + x^x \sum_{k=2}^{r+1} x^{x+\frac{r-1}{x}+\dots+\frac{1}{x}} (\log x)^r (1 + \log x) + \sum_{k=2}^{r+1} x^x \sum_{k=2}^{r+1} x^{x+\frac{r-1}{x}+\dots+\frac{k-1-1}{x}} (\log x)^{r+1-k}. \end{aligned}$$

But this expression has the same form with respect to $r+1$, that the assumption had with respect to r , and since the assumption was true for $r=2$, it is also true for all values of r greater than 2, which is what we had to prove.

Erlangen, Bayern, Hauptstrasse 83II, 26 February, 1898.

Also solved by *C. W. M. BLACK, W. W. LANDIS, and G. B. M. ZERR.*